

Mental Math

Addition and Subtraction

Teacher

If any of your students don't know their addition and subtraction facts, teach them to add and subtract using their fingers by the methods taught below. You should also reinforce basic facts using drills, games and flash cards. There are mental math strategies that make addition and subtraction easier: some effective strategies are taught in the next section. (Until your students know all their facts, allow them to add and subtract on their fingers when necessary.)

To **ADD** $4 + 8$, Grace says the greater number (8) with her fist closed. She counts up from 8, raising one finger at a time. She stops when she has raised the number of fingers equal to the lesser number (4):



She said "12" when she raised her 4th finger, so: $4 + 8 = 12$

1. Add:

- a) $5 + 2 =$ _____ b) $3 + 2 =$ _____ c) $6 + 2 =$ _____ d) $9 + 2 =$ _____
e) $2 + 4 =$ _____ f) $2 + 7 =$ _____ g) $5 + 3 =$ _____ h) $6 + 3 =$ _____
i) $11 + 4 =$ _____ j) $3 + 9 =$ _____ k) $7 + 3 =$ _____ l) $14 + 4 =$ _____
m) $21 + 5 =$ _____ n) $32 + 3 =$ _____ o) $4 + 56 =$ _____ p) $39 + 4 =$ _____

To **SUBTRACT** $9 - 5$, Grace says the lesser number (5) with her fist closed. She counts up from 5 raising one finger at a time. She stops when she says the greater number (9):



She has raised 4 fingers when she stopped, so: $9 - 5 = 4$

2. Subtract:

- a) $7 - 5 =$ _____ b) $8 - 6 =$ _____ c) $5 - 3 =$ _____ d) $5 - 2 =$ _____
e) $9 - 6 =$ _____ f) $10 - 5 =$ _____ g) $11 - 7 =$ _____ h) $17 - 14 =$ _____
i) $33 - 31 =$ _____ j) $27 - 24 =$ _____ k) $43 - 39 =$ _____ l) $62 - 58 =$ _____

Teacher

To prepare for the next section, teach your students to add 1 to any number mentally (by counting forward by 1 in their head) and to subtract 1 from any number (by counting backward by 1).

Teacher

Students who don't know how to add, subtract or estimate readily are at a great disadvantage in mathematics. Students who have trouble memorizing addition and subtraction facts can still learn to mentally add and subtract numbers in a short time if they are given daily practice in a few basic skills.

SKILL 1: Adding 2 to an Even Number

This skill has been broken down into a number of sub-skills. After teaching each sub-skill, you should give your students a short diagnostic quiz to verify that they have learned the skill. I have included sample quizzes for Skills 1 to 4.

- i) Naming the next one-digit even number:

Numbers that have ones digit 0, 2, 4, 6 or 8 are called the even numbers. Using drills or games, teach your students to say the sequence of one-digit even numbers without hesitation. Ask students to imagine the sequence going on in a circle so that the next number after 8 is 0 (0, 2, 4, 6, 8, 0, 2, 4, 6, 8...) Then play the following game: name a number in the sequence and ask your students to give the next number. Don't move on until all of your students have mastered the game.

- ii) Naming the next greatest two-digit even number:

CASE 1: Numbers that end in 0, 2, 4 or 6

Write an even two-digit number that ends in 0, 2, 4 or 6 on the board. Ask your students to name the next greatest even number. Students should recognize that if a number ends in 0, then the next even number ends in 2; if it ends in 4 then the next even number ends in 6, etc. For instance, the number 54 has ones digit 4: so the next greatest even number will have ones digit 6.

QUIZ

Name the next greatest even number:

- a) 52 : _____ b) 64 : _____ c) 36 : _____ d) 22 : _____ e) 80 : _____

CASE 2: Numbers that end in 8

Write the number 58 on the board. Ask students to name the next greatest even number. Remind your students that even numbers must end in 0, 2, 4, 6, or 8. But 50, 52, 54 and 56 are all less than 58 so the next greatest even number is 60. Your students should see that an even number ending in 8 is always followed by an even number ending in 0 (with a tens digit that is one higher).

QUIZ

Name the next greatest even number:

- a) 58 : _____ b) 68 : _____ c) 38 : _____ d) 48 : _____ e) 78 : _____

- iii) Adding 2 to an even number:

Point out to your students that adding 2 to any even number is equivalent to finding the next even number: **EXAMPLE:** $46 + 2 = 48$, $48 + 2 = 50$, etc. Knowing this, your students can easily add 2 to any even number.

QUIZ

Add:

- a) $26 + 2 = \underline{\quad}$ b) $82 + 2 = \underline{\quad}$ c) $40 + 2 = \underline{\quad}$ d) $58 + 2 = \underline{\quad}$ e) $34 + 2 = \underline{\quad}$

SKILL 2: Subtracting 2 from an Even Number

i) Finding the preceding one-digit even number:

Name a one-digit even number and ask your students to give the preceding number in the sequence. For instance, the number that comes before 4 is 2 and the number that comes before 0 is 8. (**REMEMBER:** the sequence is circular.)

ii) Finding the preceding two-digit number:

CASE 1: Numbers that end in 2, 4, 6 or 8

Write a two-digit number that ends in 2, 4, 6 or 8 on the board. Ask students to name the preceding even number. Students should recognize that if a number ends in 2, then the preceding even number ends in 0; if it ends in 4 then the preceding even number ends in 2, etc. For instance, the number 78 has ones digit 8 so the preceding even number has ones digit 6.

QUIZ

Name the preceding even number:

- a) 48 : $\underline{\quad}$ b) 26 : $\underline{\quad}$ c) 34 : $\underline{\quad}$ d) 62 : $\underline{\quad}$ e) 78 : $\underline{\quad}$

CASE 2: Numbers that end in 0

Write the number 80 on the board and ask your students to name the preceding even number. Students should recognize that if an even number ends in 0 then the preceding even number ends in 8 (but the ones digit is one less). So the even number that comes before 80 is 78.

QUIZ

Name the preceding even number:

- a) 40 : $\underline{\quad}$ b) 60 : $\underline{\quad}$ c) 80 : $\underline{\quad}$ d) 50 : $\underline{\quad}$ e) 30 : $\underline{\quad}$

ii) Subtracting 2 from an even number:

Point out to your students that subtracting 2 from an even number is equivalent to finding the preceding even number: **EXAMPLE:** $48 - 2 = 46$, $46 - 2 = 44$, etc.

QUIZ

Subtract:

- a) $58 - 2 = \underline{\quad}$ b) $24 - 2 = \underline{\quad}$ c) $36 - 2 = \underline{\quad}$ d) $42 - 2 = \underline{\quad}$ e) $60 - 2 = \underline{\quad}$

SKILL 3: Adding 2 to an Odd Number

i) Naming the next one-digit odd number:

Numbers that have ones digit 1, 3, 5, 7, and 9 are called the odd numbers. Using drills or games, teach your students to say the sequence of one-digit odd numbers without hesitation. Ask students to imagine the sequence going on in a circle so that the next number after 9 is 1 (1, 3, 5, 7, 9, 1, 3, 5, 7, 9...). Then play the following game: name a number in the sequence and ask you students to give the next number. Don't move on until all of your students have mastered the game.

ii) Naming the next greatest two-digit odd number:

CASE 1: Numbers that end in 1, 3, 5 or 7

Write an odd two-digit number that ends in 1, 3, 5, or 7 on the board. Ask you students to name the next greatest odd number. Students should recognize that if a number ends in 1, then the next even number ends in 3; if it ends in 3 then the next even number ends in 5, etc. For instance, the number 35 has ones digit 5: so the next greatest even number will have ones digit 7.

QUIZ

Name the next greatest odd number:

a) 51 : _____ b) 65 : _____ c) 37 : _____ d) 23 : _____ e) 87 : _____

CASE 2: Numbers that end in 9

Write the number 59 on the board. Ask students to name the next greatest number. Remind your students that odd numbers must end in 1, 3, 5, 7, or 9. But 51, 53, 55, and 57 are all less than 59. The next greatest odd number is 61. Your students should see that an odd number ending in 9 is always followed by an odd number ending in 1 (with a tens digit that is one higher).

QUIZ

Name the next greatest odd number:

a) 59 : _____ b) 69 : _____ c) 39 : _____ d) 49 : _____ e) 79 : _____

iii) Adding 2 to an odd number:

Point out to your students that adding 2 to any odd number is equivalent to finding the next odd number: **EXAMPLE:** $47 + 2 = 49$, $49 + 2 = 51$, etc. Knowing this, your students can easily add 2 to any odd number.

QUIZ

Add:

a) $27 + 2 =$ _____ b) $83 + 2 =$ _____ c) $41 + 2 =$ _____ d) $59 + 2 =$ _____ e) $35 + 2 =$ _____

SKILL 4: Subtracting 2 from an Odd Number

i) Finding the preceding one-digit odd number:

Name a one-digit even number and ask your students to give the preceding number in the sequence. For instance, the number that comes before 3 is 1 and the number that comes before 1 is 9. (**REMEMBER:** the sequence is circular.)

ii) Finding the preceding odd two-digit number:

CASE 1: Numbers that end in 3, 5, 7 or 9

Write a two-digit number that ends in 3, 5, 7 or 9 on the board. Ask students to name the preceding even number. Students should recognize that if a number ends in 3, then the preceding odd number ends in 1; if it ends in 5 then the preceding odd number ends in 3, etc. For instance, the number 79 has ones digit 9, so the preceding even number has ones digit 7.

QUIZ Name the preceding odd number:

a) 49 : _____ b) 27 : _____ c) 35 : _____ d) 63 : _____ e) 79 : _____

CASE 2: Numbers that end in 1

Write the number 81 on the board and ask your students to name the preceding odd number. Students should recognize that if an odd number ends in 1 then the preceding odd number ends in 9 (but the ones digit is one less). So the odd number that comes before 81 is 79.

QUIZ Name the preceding odd number:

a) 41 : _____ b) 61 : _____ c) 81 : _____ d) 51 : _____ e) 31 : _____

iii) Subtracting 2 from an odd number:

Point out to your students that subtracting 2 from an odd number is equivalent to finding the preceding even number: **EXAMPLE:** $49 - 2 = 47$, $47 - 2 = 45$, etc.

QUIZ Subtract:

a) $59 - 2 =$ _____ b) $25 - 2 =$ _____ c) $37 - 2 =$ _____ d) $43 - 2 =$ _____ e) $61 - 2 =$ _____

SKILLS 5 and 6

Once your students can add and subtract the numbers 1 and 2, then they can easily add and subtract the number 3: Add 3 to a number by first adding 2, then 1 (**EXAMPLE:** $35 + 3 = 35 + 2 + 1$). Subtract 3 from a number by subtracting 2, then subtracting 1 (**EXAMPLE:** $35 - 3 = 35 - 2 - 1$).

NOTE: All of the addition and subtraction tricks you teach your students should be reinforced with drills, flashcards and tests. Eventually students should memorize their addition and subtraction facts and shouldn't have to rely on the mental math tricks. One of the greatest gifts you can give your students is to teach them their number facts.

SKILLS 7 and 8

Add 4 to a number by adding 2 twice (**EXAMPLE:** $51 + 4 = 51 + 2 + 2$). Subtract 4 from a number by subtracting 2 twice (**EXAMPLE:** $51 - 4 = 51 - 2 - 2$).

SKILLS 9 and 10

Add 5 to a number by adding 4 then 1. Subtract 5 by subtracting 4 then 1.

SKILL 11

Students can add pairs of identical numbers by doubling (**EXAMPLE:** $6 + 6 = 2 \times 6$). Students should either memorize the 2 times table or they should double numbers by counting on their fingers by 2s.

Add a pair of numbers that differ by 1 by rewriting the larger number as 1 plus the smaller number (then use doubling to find the sum): **EXAMPLE:** $6 + 7 = 6 + 6 + 1 = 12 + 1 = 13$; $7 + 8 = 7 + 7 + 1 = 14 + 1 = 15$, etc.

SKILLS 12, 13 and 14

Add a one-digit number to 10 by simply replacing the zero in 10 by the one-digit number: **EXAMPLE:** $10 + 7 = 17$.

Add 10 to any two-digit number by simply increasing the tens digit of the two-digit number by 1: **EXAMPLE:** $53 + 10 = 63$.

Add a pair of two-digit numbers (with no carrying) by adding the ones digits of the numbers and then the tens digits: **EXAMPLE:** $23 + 64 = 87$.

SKILLS 15 and 16

To add 9 to a one-digit number, subtract 1 from the number and then add 10: **EXAMPLE:** $9 + 6 = 10 + 5 = 15$; $9 + 7 = 10 + 6 = 16$, etc. (Essentially, the student simply has to subtract 1 from the number and then stick a 1 in front of the result.)

To add 8 to a one-digit number, subtract 2 from the number and add 10: **EXAMPLE:** $8 + 6 = 10 + 4 = 14$; $8 + 7 = 10 + 5 = 15$, etc.

SKILLS 17 and 18

To subtract a pair of multiples of ten, simply subtract the tens digits and add a zero for the ones digit: **EXAMPLE:** $70 - 50 = 20$.

To subtract a pair of two-digit numbers (without carrying or regrouping), subtract the ones digit from the ones digit and the tens digit from the tens digit: **EXAMPLE:** $57 - 34 = 23$.

Mental Math

Further Strategies

Further Mental Math Strategies

1. Your students should be able to explain how to use the strategies of “rounding the subtrahend (EXAMPLE: the number you are subtracting) up to the nearest multiple of ten.”

EXAMPLES:

a) $37 - 19 = 37 - 20 + 1$ ← Subtrahend rounded to the nearest tens
 b) $64 - 28 = 64 - 30 + 2$ ← You must add 1 because 20 is 1 greater than 19
 c) $65 - 46 = 65 - 50 + 4$ ← You must add 2 because 30 is 2 greater than 28

PRACTICE QUESTIONS:

a) $27 - 17 = 27 - \underline{\quad\quad} + \underline{\quad\quad}$ d) $84 - 57 = 84 - \underline{\quad\quad} + \underline{\quad\quad}$
 b) $52 - 36 = 52 - \underline{\quad\quad} + \underline{\quad\quad}$ e) $61 - 29 = 61 - \underline{\quad\quad} + \underline{\quad\quad}$
 c) $76 - 49 = 76 - \underline{\quad\quad} + \underline{\quad\quad}$ f) $42 - 18 = 42 - \underline{\quad\quad} + \underline{\quad\quad}$

NOTE: This strategy works well with numbers that end in 6, 7, 8 or 9.

2. Your students should be able to explain how to subtract by thinking of adding.

EXAMPLES:

a) $62 - 45 = 5 + 12 = 17$ ← The sum of counting up to the nearest ten and the original number is the difference
 b) $46 - 23 = 3 + 20 = 23$
 c) $73 - 17 = 6 + 50 = 56$ ← What method did we use here?

PRACTICE QUESTIONS:

a) $88 - 36 = \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$ d) $74 - 28 = \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$
 b) $58 - 21 = \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$ e) $93 - 64 = \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$
 c) $43 - 17 = \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$ f) $82 - 71 = \underline{\quad\quad} + \underline{\quad\quad} = \underline{\quad\quad}$

3. Your students should be able to explain how to “use doubles.”

EXAMPLES:

a) $12 - 6 = 6$ ← If you add the subtrahend to itself and the sum is equal to the minuend then the subtrahend is the same as the difference
 b) $8 - 4 = 4$
 $6 + 6 = 12$ ← Same value as minuend
 Minuend plus itself

PRACTICE QUESTIONS:

a) $6 - 3 = \underline{\quad\quad}$ d) $18 - 9 = \underline{\quad\quad}$
 b) $10 - 5 = \underline{\quad\quad}$ e) $16 - 8 = \underline{\quad\quad}$
 c) $14 - 7 = \underline{\quad\quad}$ f) $20 - 10 = \underline{\quad\quad}$

Mental Math Exercises

NOTE TO TEACHER: Teaching the material on these worksheets may take several lessons. Students will need more practice than is provided on these pages. These pages are intended as a test to be given when you are certain your students have learned the materials fully.

Teacher

Teach **SKILLS 1, 2, 3 AND 4** as outlined on pages 2 through 5 before you allow your students to answer Questions 1 through 12:

- Name the even number that comes after the number. Answer in the blank provided:
a) 32 _____ b) 46 _____ c) 14 _____ d) 92 _____ e) 56 _____
f) 30 _____ g) 84 _____ h) 60 _____ i) 72 _____ j) 24 _____
- Name the even number that comes after the number:
a) 28 _____ b) 18 _____ c) 78 _____ d) 38 _____ e) 68 _____
- Add. **REMEMBER:** adding 2 to an even number is the same as finding the next even number:
a) $42 + 2 =$ _____ b) $76 + 2 =$ _____ c) $28 + 2 =$ _____ d) $16 + 2 =$ _____
e) $68 + 2 =$ _____ f) $12 + 2 =$ _____ g) $36 + 2 =$ _____ h) $90 + 2 =$ _____
i) $70 + 2 =$ _____ j) $24 + 2 =$ _____ k) $66 + 2 =$ _____ l) $52 + 2 =$ _____
- Name the even number that comes before the number:
a) 38 _____ b) 42 _____ c) 56 _____ d) 72 _____ e) 98 _____
f) 48 _____ g) 16 _____ h) 22 _____ i) 66 _____ j) 14 _____
- Name the even number that comes before the number:
a) 30 _____ b) 70 _____ c) 60 _____ d) 10 _____ e) 80 _____
- Subtract. **REMEMBER:** subtracting 2 from an even number is the same as finding the preceding even number:
a) $46 - 2 =$ _____ b) $86 - 2 =$ _____ c) $90 - 2 =$ _____ d) $14 - 2 =$ _____
e) $54 - 2 =$ _____ f) $72 - 2 =$ _____ g) $12 - 2 =$ _____ h) $56 - 2 =$ _____
i) $32 - 2 =$ _____ j) $40 - 2 =$ _____ k) $60 - 2 =$ _____ l) $26 - 2 =$ _____
- Name the odd number that comes after the number:
a) 37 _____ b) 51 _____ c) 63 _____ d) 75 _____ e) 17 _____
f) 61 _____ g) 43 _____ h) 81 _____ i) 23 _____ j) 95 _____
- Name the odd number that comes after the number:
a) 69 _____ b) 29 _____ c) 9 _____ d) 79 _____ e) 59 _____

Exercises

9. Add. **REMEMBER:** Adding 2 to an odd number is the same as finding the next odd number:
- a) $25 + 2 =$ _____ b) $31 + 2 =$ _____ c) $47 + 2 =$ _____ d) $33 + 2 =$ _____
e) $39 + 2 =$ _____ f) $91 + 2 =$ _____ g) $5 + 2 =$ _____ h) $89 + 2 =$ _____
i) $11 + 2 =$ _____ j) $65 + 2 =$ _____ k) $29 + 2 =$ _____ l) $17 + 2 =$ _____
10. Name the odd number that comes before the number:
- a) 39 _____ b) 43 _____ c) 57 _____ d) 17 _____ e) 99 _____
f) 13 _____ g) 85 _____ h) 79 _____ i) 65 _____ j) 77 _____
11. Name the odd number that comes before the number:
- a) 21 _____ b) 41 _____ c) 11 _____ d) 91 _____ e) 51 _____
12. Subtract. **REMEMBER:** Subtracting 2 from an odd number is the same as finding the preceding odd number.
- a) $47 - 2 =$ _____ b) $85 - 2 =$ _____ c) $91 - 2 =$ _____ d) $15 - 2 =$ _____
e) $51 - 2 =$ _____ f) $73 - 2 =$ _____ g) $11 - 2 =$ _____ h) $59 - 2 =$ _____
i) $31 - 2 =$ _____ j) $43 - 2 =$ _____ k) $7 - 2 =$ _____ l) $25 - 2 =$ _____

Teacher

Teach **SKILLS 5 AND 6** as outlined on pages 5 and 6 before you allow your students to answer Questions 13 and 14:

13. Add 3 to the number by adding 2, then adding 1 (**EXAMPLE:** $35 + 3 = 35 + 2 + 1$):
- a) $23 + 3 =$ _____ b) $36 + 3 =$ _____ c) $29 + 3 =$ _____ d) $16 + 3 =$ _____
e) $67 + 3 =$ _____ f) $12 + 3 =$ _____ g) $35 + 3 =$ _____ h) $90 + 3 =$ _____
i) $78 + 3 =$ _____ j) $24 + 3 =$ _____ k) $6 + 3 =$ _____ l) $59 + 3 =$ _____
14. Subtract 3 from the number by subtracting 2, then subtracting 1 (**EXAMPLE:** $35 - 3 = 35 - 2 - 1$):
- a) $46 - 3 =$ _____ b) $87 - 3 =$ _____ c) $99 - 3 =$ _____ d) $14 - 3 =$ _____
e) $8 - 3 =$ _____ f) $72 - 3 =$ _____ g) $12 - 3 =$ _____ h) $57 - 3 =$ _____
i) $32 - 3 =$ _____ j) $40 - 3 =$ _____ k) $60 - 3 =$ _____ l) $28 - 3 =$ _____
15. Fred has 49 stamps. He gives 2 stamps away. How many stamps does he have left?
16. There are 25 minnows in a tank. Alice adds 3 more to the tank. How many minnows are now in the tank?

Teacher

Teach **SKILLS 7 AND 8** as outlined on page 6.

17. Add 4 to the number by adding 2 twice (**EXAMPLE:** $51 + 4 = 51 + 2 + 2$):

- a) $42 + 4 =$ _____ b) $76 + 4 =$ _____ c) $27 + 4 =$ _____ d) $17 + 4 =$ _____
 e) $68 + 4 =$ _____ f) $11 + 4 =$ _____ g) $35 + 4 =$ _____ h) $8 + 4 =$ _____
 i) $72 + 4 =$ _____ j) $23 + 4 =$ _____ k) $60 + 4 =$ _____ l) $59 + 4 =$ _____

18. Subtract 4 from the number by subtracting 2 twice (**EXAMPLE:** $26 - 4 = 26 - 2 - 2$):

- a) $46 - 4 =$ _____ b) $86 - 4 =$ _____ c) $91 - 4 =$ _____ d) $15 - 4 =$ _____
 e) $53 - 4 =$ _____ f) $9 - 4 =$ _____ g) $13 - 4 =$ _____ h) $57 - 4 =$ _____
 i) $40 - 4 =$ _____ j) $88 - 4 =$ _____ k) $69 - 4 =$ _____ l) $31 - 4 =$ _____

Teacher

Teach **SKILLS 9 AND 10** as outlined on page 6.

19. Add 5 to the number by adding 4, then adding 1 (or add 2 twice, then add 1):

- a) $84 + 5 =$ _____ b) $27 + 5 =$ _____ c) $31 + 5 =$ _____ d) $44 + 5 =$ _____
 e) $63 + 5 =$ _____ f) $92 + 5 =$ _____ g) $14 + 5 =$ _____ h) $16 + 5 =$ _____
 i) $9 + 5 =$ _____ j) $81 + 5 =$ _____ k) $51 + 5 =$ _____ l) $28 + 5 =$ _____

20. Subtract 5 from the number by subtracting 4, then subtracting 1 (or subtract 2 twice, then subtract 1):

- a) $48 - 5 =$ _____ b) $86 - 5 =$ _____ c) $55 - 5 =$ _____ d) $69 - 5 =$ _____
 e) $30 - 5 =$ _____ f) $13 - 5 =$ _____ g) $92 - 5 =$ _____ h) $77 - 5 =$ _____
 i) $45 - 5 =$ _____ j) $24 - 5 =$ _____ k) $91 - 5 =$ _____ l) $8 - 5 =$ _____

Teacher

Teach **SKILLS 11** as outlined on page 6.

21. Add:

- a) $6 + 6 =$ _____ b) $7 + 7 =$ _____ c) $8 + 8 =$ _____
 d) $5 + 5 =$ _____ e) $4 + 4 =$ _____ f) $9 + 9 =$ _____

22. Add by thinking of the larger number as a sum of two smaller numbers:

- a) $6 + 7 = 6 + 6 + 1$ b) $7 + 8 =$ _____ c) $6 + 8 =$ _____
 d) $4 + 5 =$ _____ e) $5 + 7 =$ _____ f) $8 + 9 =$ _____

Mental Math Practice Sheet

Teacher

Teach **SKILLS 12, 13 AND 14** as outlined on page 6.

23. a) $10 + 3 =$ _____ b) $10 + 7 =$ _____ c) $5 + 10 =$ _____ d) $10 + 1 =$ _____
e) $9 + 10 =$ _____ f) $10 + 4 =$ _____ g) $10 + 8 =$ _____ h) $10 + 2 =$ _____
24. a) $10 + 20 =$ _____ b) $40 + 10 =$ _____ c) $10 + 80 =$ _____ d) $10 + 50 =$ _____
e) $30 + 10 =$ _____ f) $10 + 60 =$ _____ g) $10 + 10 =$ _____ h) $70 + 10 =$ _____
25. a) $10 + 25 =$ _____ b) $10 + 67 =$ _____ c) $10 + 31 =$ _____ d) $10 + 82 =$ _____
e) $10 + 43 =$ _____ f) $10 + 51 =$ _____ g) $10 + 68 =$ _____ h) $10 + 21 =$ _____
i) $10 + 11 =$ _____ j) $10 + 19 =$ _____ k) $10 + 44 =$ _____ l) $10 + 88 =$ _____
26. a) $20 + 30 =$ _____ b) $40 + 20 =$ _____ c) $30 + 30 =$ _____ d) $50 + 30 =$ _____
e) $20 + 50 =$ _____ f) $40 + 40 =$ _____ g) $50 + 40 =$ _____ h) $40 + 30 =$ _____
i) $60 + 30 =$ _____ j) $20 + 60 =$ _____ k) $20 + 70 =$ _____ l) $60 + 40 =$ _____
27. a) $20 + 23 =$ _____ b) $32 + 24 =$ _____ c) $51 + 12 =$ _____ d) $12 + 67 =$ _____
e) $83 + 14 =$ _____ f) $65 + 24 =$ _____ g) $41 + 43 =$ _____ h) $70 + 27 =$ _____
i) $31 + 61 =$ _____ j) $54 + 33 =$ _____ k) $28 + 31 =$ _____ l) $42 + 55 =$ _____

Teacher

Teach **SKILLS 15 AND 16** as outlined on page 6.

28. a) $9 + 3 =$ _____ b) $9 + 7 =$ _____ c) $6 + 9 =$ _____ d) $4 + 9 =$ _____
e) $9 + 9 =$ _____ f) $5 + 9 =$ _____ g) $9 + 2 =$ _____ h) $9 + 8 =$ _____
29. a) $8 + 2 =$ _____ b) $8 + 6 =$ _____ c) $8 + 7 =$ _____ d) $4 + 8 =$ _____
e) $5 + 8 =$ _____ f) $8 + 3 =$ _____ g) $9 + 8 =$ _____ h) $8 + 8 =$ _____

Teacher

Teach **SKILLS 17 AND 18** as outlined on page 6.

30. a) $40 - 10 =$ _____ b) $50 - 10 =$ _____ c) $70 - 10 =$ _____ d) $20 - 10 =$ _____
e) $40 - 20 =$ _____ f) $60 - 30 =$ _____ g) $40 - 30 =$ _____ h) $60 - 50 =$ _____
31. a) $57 - 34 =$ _____ b) $43 - 12 =$ _____ c) $62 - 21 =$ _____ d) $59 - 36 =$ _____
e) $87 - 63 =$ _____ f) $95 - 62 =$ _____ g) $35 - 10 =$ _____ h) $17 - 8 =$ _____

Mental Math

Advanced

Multiples of Ten

STUDENT: In the exercises below, you will learn several ways to use multiples of ten in mental addition or subtraction.

EXAMPLE 1

$$542 + 214 = 542 + 200 + 10 + 4 = 742 + 10 + 4 = 752 + 4 = 756$$

$$827 - 314 = 827 - 300 - 10 - 4 = 527 - 10 - 4 = 517 - 4 = 713$$

Sometimes you will need to carry:

$$545 + 172 = 545 + 100 + 70 + 2 = 645 + 70 + 2 = 715 + 2 = 717$$

1. Warm up:

a) $536 + 100 = \underline{\hspace{2cm}}$ b) $816 + 10 = \underline{\hspace{2cm}}$ c) $124 + 5 = \underline{\hspace{2cm}}$ d) $540 + 200 = \underline{\hspace{2cm}}$

e) $234 + 30 = \underline{\hspace{2cm}}$ f) $345 + 300 = \underline{\hspace{2cm}}$ g) $236 - 30 = \underline{\hspace{2cm}}$ h) $442 - 20 = \underline{\hspace{2cm}}$

i) $970 - 70 = \underline{\hspace{2cm}}$ j) $542 - 400 = \underline{\hspace{2cm}}$ k) $160 + 50 = \underline{\hspace{2cm}}$ l) $756 + 40 = \underline{\hspace{2cm}}$

2. Write the second number in expanded form and add or subtract one digit at a time. The first one is done for you:

a) $564 + 215 = \underline{564 + 200 + 10 + 5} = \underline{779}$

b) $445 + 343 = \underline{\hspace{4cm}} = \underline{\hspace{2cm}}$

c) $234 + 214 = \underline{\hspace{4cm}} = \underline{\hspace{2cm}}$

3. Add or subtract mentally (one digit at a time):

a) $547 + 312 = \underline{\hspace{2cm}}$ b) $578 - 314 = \underline{\hspace{2cm}}$ c) $845 - 454 = \underline{\hspace{2cm}}$

EXAMPLE 2

If one of the numbers you are adding or subtracting is close to a number with a multiple of ten, add the multiple of ten and then add or subtract an adjustment factor:

$$645 + 99 = 645 + 100 - 1 = 745 - 1 = 744$$

$$856 + 42 = 856 + 40 + 2 = 896 + 2 = 898$$

EXAMPLE 3

Sometimes in subtraction, it helps to think of a multiple of ten as a sum of 1 and a number consisting entirely of 9s (**EXAMPLE:** $100 = 1 + 99$; $1000 = 1 + 999$). You never have to borrow or exchange when you are subtracting from a number consisting entirely of 9s.

$$100 - 43 = 1 + 99 - 43 = 1 + 56 = 57 \quad \leftarrow \text{Do the subtraction, using 99 instead of 100, and then add 1 to your answer.}$$

$$1000 - 543 = 1 + 999 - 543 = 1 + 456 = 457$$

4. Use the tricks you've just learned:

a) $845 + 91 = \underline{\hspace{2cm}}$ b) $456 + 298 = \underline{\hspace{2cm}}$ c) $100 - 84 = \underline{\hspace{2cm}}$ d) $1000 - 846 = \underline{\hspace{2cm}}$

Mental Math

Game: Modified Go Fish

Purpose

If students know the pairs of one-digit numbers that add up to particular target numbers, they will be able to mentally break sums into easier sums.

EXAMPLE: As it is easy to add any one-digit number to 10, you can add a sum more readily if you can decompose numbers in the sum into pairs that add to ten.

$$7 + 5 = \underbrace{7 + 3} + 2 = 10 + 2 = 12$$

These numbers add to 10.

To help students remember pairs of numbers that add up to a given target number I developed a variation of “Go Fish” that I have found very effective.

The Game

Pick any target number and remove all the cards with value greater than or equal to the target number out of the deck. In what follows, I will assume that the target number is 10, so you would take all the tens and face cards out of the deck (Aces count as one).

The dealer gives each player 6 cards. If a player has any pairs of cards that add to 10 they are allowed to place these pairs on the table before play begins.

Player 1 selects one of the cards in his or her hand and asks the Player 2 for a card that adds to 10 with the chosen card. For instance, if Player 1’s card is a 3, they may ask the Player 2 for a 7.

If Player 2 has the requested card, the first player takes it and lays it down along with the card from their hand. The first player may then ask for another card. If the Player 2 doesn’t have the requested card they say: “Go fish,” and the Player 1 must pick up a card from the top of the deck. (If this card adds to 10 with a card in the player’s hand they may lay down the pair right away). It is then Player 2’s turn to ask for a card.

Play ends when one player lays down all of their cards. Players receive 4 points for laying down all of their cards first and 1 point for each pair they have laid down.

NOTE: With weaker students I would recommend you start with pairs of numbers that add to 5. Take all cards with value greater than 4 out of the deck. Each player should be dealt only 4 cards to start with.

I have worked with several students who have had a great deal of trouble sorting their cards and finding pairs that add to a target number. I’ve found the following exercise helps:

Give your student only three cards; two of which add to the target number. Ask the student to find the pair that add to the target number. After the student has mastered this step with 3 cards repeat the exercise with 4 cards, then 5 cards, and so on.

NOTE: You can also give your student a list of pairs that add to the target number. As the student gets used to the game, gradually remove pairs from the list so that the student learns the pairs by memory.

Mental Math Checklist #1

Student Name	Can Add 1 to Any Number	Can Subtract 1 from Any Number	Can Add 2 to Any Number	Can Subtract 2 from Any Number	Knows All Pairs that Add to 5	Can Double 1-Digit Numbers

Mental Math Checklist #2

Student Name	Can Add Near Doubles. EXAMPLE: $6 + 7 =$ $6 + 6 + 1$	Can Add a 1-Digit Number to Any Multiple of 10. EXAMPLE: $30 + 6 = 36$	Can Add Any 1-Digit Number to a Number Ending in 9. EXAMPLE: $29 + 7 =$ $30 + 6 = 36$	Can Add 1-Digit Numbers by "Breaking" them Apart into Pairs that Add to 10. EXAMPLE: $7 + 5 =$ $7 + 3 + 2 =$ $10 + 2$	Can Subtract Any Multiple of 10 from 100. EXAMPLE: $100 - 40 = 60$

Mental Math Checklist #3

Student Name	Can Mentally Make Change from a Dollar. SEE: Workbook Sheets on Money.	Can Mentally Add Any Pair of 1-Digit Numbers.	Can Mentally Subtract Any Pair of 1-Digit Numbers.	Student Can Multiply and Count by:								
				2	3	4	5	6	7	8	9	

Mental Math

How to Learn Your Times Tables in a Week

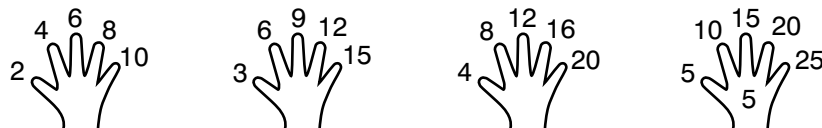
Teacher

Trying to do math without knowing your times tables is like trying to play the piano without knowing the location of the notes on the keyboard. Your students will have difficulty seeing patterns in sequences and charts, solving proportions, finding equivalent fractions, decimals and percents, solving problems etc. if they don't know their tables.

Using the method below, you can teach your students their tables in a week or so. (If you set aside five or ten minutes a day to work with students who need extra help, the pay-off will be enormous.) There is really no reason for your students not to know their tables!

DAY 1: Counting by 2s, 3s, 4s and 5s

If you have completed the Fractions Unit you should already know how to count and multiply by 2s, 3s, 4s and 5s. If you don't know how to count by these numbers you should memorize the hands below:



If you know how to count by 2s, 3s, 4s and 5s, then you can multiply by any combination of these numbers. For instance, to find the product 3×2 , count by 2s until you have raised 3 fingers.



DAY 2: The Nine Times Table

The numbers you say when you count by 9s are called the **MULTIPLES** of 9 (zero is also a multiple of 9). The first ten multiples of 9 (after zero) are: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90. What happens when you add the digits of any of these multiples of 9 (**EXAMPLE:** $1 + 8$ or $6 + 3$)? The sum is always 9!

Here is another useful fact about the nine times table: Multiply 9 by any number between 1 and 10 and look at the tens digit of the product. The tens digit is always one less than the number you multiplied by:

$$\begin{array}{ccc} 9 \times 4 = 36 & 9 \times 8 = 72 & 9 \times 2 = 18 \\ \uparrow & \uparrow & \uparrow \\ 3 \text{ is one less than } 4 & 7 \text{ is one less than } 8 & 1 \text{ is one less than } 2 \end{array}$$

You can find the product of 9 and any number by using the two facts given above. For instance, to find 9×7 , follow these steps:

$$\begin{array}{ccc} \text{STEP 1:} & 9 \times 7 = \underline{\quad} \underline{\quad} & 9 \times 7 = \underline{\quad} \underline{\quad} \\ \uparrow & \uparrow & \uparrow \\ \text{Subtract 1 from the number} & & \text{Now you know the} \\ \text{you are multiplying by 9: } 7 - 1 = 6 & & \text{tens digit of the product.} \end{array}$$

Teacher

1. Make sure your students know how to subtract (by counting on their fingers if necessary) before you teach them the trick for the nine times table.
2. Give a test on **STEP 1** (above) before you move on.

STEP 2: $9 \times 7 = \underline{\quad} \underline{\quad}$

 ↑ ↑
These two digits add to 9.

$9 \times 7 = \underline{\quad} \underline{\quad}$

 ↑
*So the missing digit is $9 - 6 = 3$
(You can do the subtraction on your fingers if necessary).*

Practise these two steps for all of the products of 9: 9×2 , 9×3 , 9×4 , etc.

DAY 3: The Eight Times Table

There are two patterns in the digits of the 8 times table. Knowing these patterns will help you remember how to count by 8s.

STEP 1: You can find the ones digit of the first five multiples of 8, by starting at 8 and counting backwards by 2s.

- 8
- 6
- 4
- 2
- 0

STEP 2: You can find the tens digit of the first five multiples of 8, by starting at 0 and count up by 1s.

- 08
- 16
- 24
- 32
- 40

(Of course you don't need to write the 0 in front of the 8 for the product 1×8 .)

STEP 3: You can find the ones digit of the next five multiples of 8 by repeating step 1:

- 8
- 6
- 4
- 2
- 0

STEP 4: You can find the remaining tens digits by starting at 4 and count up by 1s.

- 48
- 56
- 64
- 72
- 80

Practise writing the multiples of 8 (up to 80) until you have memorized the complete list. Knowing the patterns in the digits of the multiples of 8 will help you memorize the list very quickly. Then you will know how to multiply by 8:

$8 \times 6 = 48$

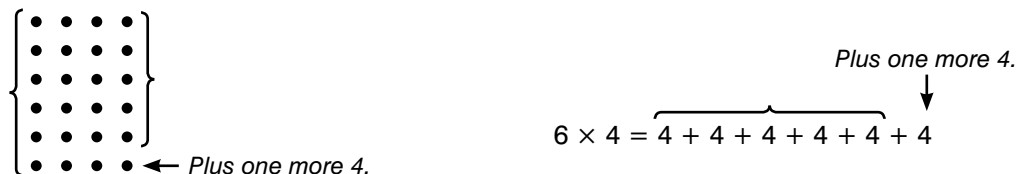


Count by eight until you have 6 fingers up: 8, 16, 24, 32, 40, 48.

DAY 4: The Six Times Table

If you have learned the eight and nine times tables, then you already know 6×9 and 6×8 .

And if you know how to multiply by 5 up to 5×5 , then you also know how to multiply by 6 up to 6×5 ! That's because you can always calculate 6 times a number by calculating 5 times the number and then adding the number itself to the result. The pictures below show why this works for 6×4 :



$$6 \times 4 = 5 \times 4 + 4 = 20 + 4 = 24$$

Similarly:

$$6 \times 2 = 5 \times 2 + 2; \quad 6 \times 3 = 5 \times 3 + 3; \quad 6 \times 5 = 5 \times 5 + 5.$$

Knowing this, you only need to memorize 2 facts:

ONE: $6 \times 6 = 36$

TWO: $6 \times 7 = 42$

Or, if you know 6×5 , you can find 6×6 by calculating $6 \times 5 + 6$.

DAY 5: The Seven Times Table

If you have learned the six, eight and nine times tables, then you already know:

$$6 \times 7, 8 \times 7 \text{ and } 9 \times 7.$$

And since you also already know $1 \times 7 = 7$, you only need to memorize 5 facts:

$$1. 2 \times 7 = 14 \quad 2. 3 \times 7 = 21 \quad 3. 4 \times 7 = 28 \quad 4. 5 \times 7 = 35 \quad 5. 7 \times 7 = 49$$

If you are able to memorize your own phone number, then you can easily memorize these 5 facts!

NOTE: You can use doubling to help you learn the facts above. 4 is double 2, so $4 \times 7 (= 28)$ is double $2 \times 7 (= 14)$. 6 is double 3, so $6 \times 7 (= 42)$ is double $3 \times 7 (= 21)$.

Try this test every day until you have learned your times tables:

- | | | | |
|--------------------------|--------------------------|--------------------------|--------------------------|
| 1. $3 \times 5 =$ _____ | 2. $8 \times 4 =$ _____ | 3. $9 \times 3 =$ _____ | 4. $4 \times 5 =$ _____ |
| 5. $2 \times 3 =$ _____ | 6. $4 \times 2 =$ _____ | 7. $8 \times 1 =$ _____ | 8. $6 \times 6 =$ _____ |
| 9. $9 \times 7 =$ _____ | 10. $7 \times 7 =$ _____ | 11. $5 \times 8 =$ _____ | 12. $2 \times 6 =$ _____ |
| 13. $6 \times 4 =$ _____ | 14. $7 \times 3 =$ _____ | 15. $4 \times 9 =$ _____ | 16. $2 \times 9 =$ _____ |
| 17. $9 \times 9 =$ _____ | 18. $3 \times 4 =$ _____ | 19. $6 \times 8 =$ _____ | 20. $7 \times 5 =$ _____ |
| 21. $9 \times 5 =$ _____ | 22. $5 \times 6 =$ _____ | 23. $6 \times 3 =$ _____ | 24. $7 \times 1 =$ _____ |
| 25. $8 \times 3 =$ _____ | 26. $9 \times 6 =$ _____ | 27. $4 \times 7 =$ _____ | 28. $3 \times 3 =$ _____ |
| 29. $8 \times 7 =$ _____ | 30. $1 \times 5 =$ _____ | 31. $7 \times 6 =$ _____ | 32. $2 \times 8 =$ _____ |